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REGULAR PAPER

Time-delayed dual-rate haptic rendering: stability analysis and reduced order modeling

Suhail Ganiny[1] · Majid H. Koul[2] · Babar Ahmad[1]

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Abstract
This work presents an exhaustive analysis of the impact of time delay on the stability of dual-rate haptics controllers using an exact discrete-time method. The mathematical formulation of such controllers leads to higher order state-space models, in particular, for higher values of time delay and sampling rates. A balanced truncation based model order reduction framework is therefore utilized for obtaining reduced order models, while preserving the stability properties of the original higher order models. The likely order of the reduced models is selected on the basis of the Hankel singular values which represent the contribution of the system states towards the overall system energy. Using this framework, it is empirically found that third order reduced models yield exactly the same stability ranges as given by the allied full order models. This empirical finding is backed up by a rigorous analysis of the controller while considering a wide range of values for the time delay spanning both the realistic and worst-case application scenarios. This result is hitherto unknown in the haptics literature and is for the first time reported in this paper. For comparison purposes, the stability ranges of the dual-rate controller are also obtained using an equivalent continuous-time method, and numerical simulations. This work generalizes the results of previous works available in the literature for uniform-rate sampling scheme both for delayed and non-delayed haptics controllers. The results demonstrate that for a time-delayed dual-rate haptics controller, an increase in the sampling rate leads to an enhancement in the stable range of virtual wall parameters as long as the value of time delay relative to the sampling rate (non-dimensional time delay) remains small. For higher values of the non-dimensional time delay, higher sampling rates do not necessarily lead to performance enhancement.

Keywords Dual-rate haptics controllers · Time delay · State-space modeling · Model order reduction · Balanced truncation · Hankel singular values · Impedance haptic interfaces

1 Introduction

Haptic interfaces enable a human operator to simultaneously perceive and to act upon a virtual (or distant) environment using the sense of touch. These interfaces make physical interactions with the virtual objects feel more natural and realistic and have thereby found widespread applications in a multitude of domains, for instance in the medical field (Wang et al. 2017), education (Okamura et al. 2002), training (Abate et al. 2009), entertainment (Mazzoni and Bryan-Kinns 2016), virtual prototyping (Ix et al. 2001), forensics (Buck et al. 2008) etc.

A haptic interface typically provides kinesthetic feedback to a user wherein a force proportional to the impedance of a virtual environment is applied, in response to an admissible motion input. It is well known (Colgate and Brown 1994; Diolaiti et al. 2006), that attempts made to emulate stiff virtual environments often lead to device instability, yet rendering high stiffness is desirable because of human perception thresholds (Tan et al. 1994), and practical considerations (Wang et al. 2011). Factors that are responsible for this inadequacy of haptic devices have been identified in Minsky et al. (1990); Colgate and Brown (1994), Diolaiti

✉ Suhail Ganiny
suhail_15phd16@nitsri.ac.in

Majid H. Koul
majidkoul@gmail.com

Babar Ahmad
babar@nitsri.ac.in

1 Department of Mechanical Engineering, National Institute of Technology Srinagar, Srinagar, J&K 190006, India

2 Department of Mechanical Engineering, Islamic University of Science and Technology, Awantipora, J&K 192122, India