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## Exact discrete-time stability analysis of multi-DOF haptic rendering: Impact of multi-rate, time-delay, and mechanical parameters

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## Abstract

Previous studies on the stability analysis of haptic devices have predominantly focused on single-DOF devices, thereby limiting attention to multi-DOF devices, particularly those employing multi-rate sampling schemes. In this paper, we introduce a formulation for the coupled dynamics between the haptic device and the virtual environment for a multi-DOF haptic device controlled using a dual-rate sampling scheme. Subsequently, we analyze its stability through the application of a dynamic decoupling strategy within an exact discrete-time state-space framework while the device is engaged in rendering a virtual wall along one of its operational space coordinates. Furthermore, we explore how the combined influence of the dual-rate sampling approach, time delay, and the mechanical design affects the stability boundaries of the multi-DOF haptic device at a fixed workspace location as well as within the entire usable workspace. Additionally, we utilize a model-order reduction (MOR) framework to simplify the determination of the device's stability limits, irrespective of the specific combinations of time delay and sampling rates employed.

## **Keywords**

Haptic interface, dual-rate sampling, multi-DOF, stability, Z-width, model-order reduction

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## Introduction

Human-machine interfaces (HMIs), such as headmounted displays, headsets, body suits, and haptic devices, form an integral part of teleoperated and extended reality systems by enabling information exchange between the rendered environment and the user.<sup>1</sup> Among these HMIs, haptic devices facilitate bidirectional interaction through the sense of touch, conveying information about the geometrical, inertial, and material properties of remote or virtual objects.<sup>2</sup> The incorporation of haptic-based feedback in virtual and remote interactions offers significant benefits, such as reducing cutting forces in remote or virtual surgery,<sup>3</sup> assisting sound localization in hearingimpaired individuals,<sup>4</sup> and enhancing the sense of immersion.<sup>5</sup>

Haptic devices have been designed across a spectrum, spanning from expansive full-body exoskeletons to compact fingertip-sized tactile devices.<sup>6,7</sup> However, the most widely adopted category in both research and commercial spheres continues to be the grounded kinesthetic devices with force feedback, known as *impedance-type* devices.<sup>8</sup> For a realistic portrayal of virtual or remote environments, these devices are required to possess a wider mechanical bandwidth, often referred to as the *Z*-width.<sup>9,10</sup> From a structural standpoint, meeting this demand necessitates that haptic devices exhibit minimal inertia and high structural stiffness.<sup>11</sup> Haptic devices utilizing a parallel kinematic architecture excel in meeting these prerequisites, making them prevalent in various applications. For instance, Koehler et al.<sup>12</sup> have very recently developed a 5-DOF parallel haptic device with compliant linkages, while Williams et al.<sup>13</sup> have designed a finger-mounted 4-DOF device using origami principles. Simpler multi-DOF parallel haptic devices, such as the 2-DOF 5 bar mechanism capable of providing force feedback within a planar workspace, have found applications in various fields.<sup>14</sup> Such devices have

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